To the Editor:

Kendoush (1994) implies that the expression derived by Lochiel and Calderbank (1964) [his Eq. 6] for the mean Sherwood number over the frontal surface of a spherical bubble cap in potential flow is in error. He demonstrates this presumed error by plotting Eq. 6 together with his own expression (Eq. 46) in his Figure 6. He cites Weber (1975) as attributing a 10–15% deviation of the expression of Lochiel and Calderbank from experimental data to "the incorrect use of the velocity component in the steady-state mass diffusion equation" and implies this is the source of the discrepancy in Figure 6.

Both the implication of an error and its attribution are wrong. First, the expression of Lochiel and Calderbank is exact for potential flow over a spherical bubble cap for asymptotically large Reynolds numbers such that diffusion in the direction of the streamlines and the effects of transverse curvature are negligible. The "new" expression of Kendoush differs only in that he (arbitrarily) uses the radius of curvature as the characteristic dimension in the Sherwood and Peclet numbers rather than the volume-equivalent diameter used by Lochiel and Calderbank. If the ratio of these characteristic lengths

$$\frac{D_{\text{curvature}}}{D_{\text{equivalent}}} = \frac{4+E^2}{2^{4/3}(4+3E^2)^{1/3}}$$

is introduced, all of the expressions of Kendoush for a spherical cap follow directly from those of Lochiel and Calderbank.

Second, Weber (1975) demonstrated that the 10–15% deviation of experimental data for whole bubbles from an expression of Lochiel and Calderbank for whole bubbles is due to the deviation of the fluid motion from potential flow for finite ratios of the viscosity of the gas to that of the liquid. This same criticism is equally applicable to all of the expressions of Kendoush.

In summary, the results of Kendoush for the frontal portion of a spherical bubble cap are merely reexpressions, not corrections, for those of Lochiel and Calderbank. They are subject to exactly the same limitations. The speculative results of Kendoush for the rear of the spherical cap appear to be new.

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Reply:

I am grateful to Prof. Churchill for his interest in my article (1994, p. 1440).

J. Y. Parlange (*J. Fluid Mech.*, 1969, p. 257), Coppus and Rietema (*Chem. Eng. Sci.*, 1980, p. 1497), and Ryskin and Leal (*J. Fluid Mech.*, 1984, p. 19), among others, all have used the radius of curvature of the spherical-cap bubble as a characteristic length in their dimensionless numbers. Accordingly, my approach was not taken "arbitrarily" by doing the same.

My results for the frontal portion of the spherical-cap bubble are not "reexpressions" of Lochiel and Calderbank (1964), since Eq. 46 of my article is

$$Nu = 1.13(Pe)^{1/2}\left(\frac{3E^2+4}{E^2+4}\right)^{1/2}$$

To base the dimensionless numbers of the above equation on the equivalent diameter (d) we get:

$$Nu^* = \frac{hd}{k}$$

$$= 1.13\left(\frac{d}{2a}\right)^{1/2}(Pe^*)^{1/2}\left(\frac{3E^2+4}{E^2+4}\right)^{1/2} \quad \text{(i)}$$

where $Pe^* = Ud/\alpha$

Introducing the equation derived by Churchill for $(d/2a)$ to the above equation we get

$$Nu^* = 1.79(3E^2+4)^{-1/12}(Pe^*)^{1/2}$$

which is not similar to the equation derived by Lochiel and Calderbank as

$$Nu^* = 1.79\frac{(3E^2+4)^{2/3}}{E^2+4}(Pe^*)^{1/2} \quad \text{(ii)}$$

Even if we use a more direct approach of inserting the relationship

$$2a = 2.4d$$

to Eq. i we get

$$Nu^* = 0.745\left(\frac{3E^2+4}{E^2+4}\right)^{1/2}(Pe^*)^{1/2}$$

which is again not similar to Eq. ii of Lochiel and Calderbank.

The volume of the spherical-cap bubble is

$$H = \pi a^3(2/3 - \cos\theta_m + 1/3\cos^3\theta_m)$$
$$= (1/6)\pi d^3$$

for $\theta_m = 48$ deg

$$H = \pi a^3(0.0974) = (1/6)\pi d^3$$

hence

$$2a = 2.4d$$

There are fundamental differences between my equations and those of Lochiel and Calderbank, for example

- Lochiel and Calderbank's equation does not reduce to the circular disk solution when $E \to \infty$ in Eq. ii as we get $Nu = 3.72\ Pe^{1/2}$ while my Eq. 46 does so as explained in the article.

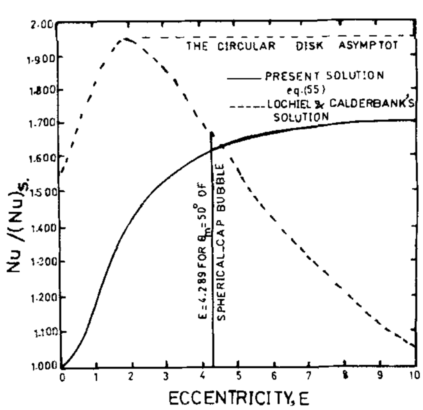

Figure 6. Present solution vs. that of Lochiel and Calderbank (1964).

• Equation ii of Lochiel and Calderbank does not reduce to the single sphere solution upon letting $E = 0$, while my Eq. 46 does.

• The solution of Lochiel and Calderbank does not provide a distribution of the heat-transfer rate over the surfaces of the cap as given in my Eq. 41 and Figure 3 of the article.

• The difference between the characteristic length of the dimensionless numbers of my equations and those of Lochiel and Calderbank needs correcting Figure 6 of the article to the one shown, which still shows the difference between my solution and that of Lochiel and Calderbank; however, the equation of Lochiel and Calderbank becomes

$$Nu/(Nu)_s = 2.454 \frac{(3E^2+4)^{2/3}}{E^2+4}$$

when the dimensionless numbers were based on the curvature of the spherical-cap bubble.

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BOOK REVIEWS

Applied Fluid Mechanics

By T. C. Papanastasiou, Prentice-Hall, Englewood Cliffs, NJ, 1994, 520 pp.

This book is intended as a textbook for a first course in fluid mechanics at the undergraduate level. There is an emphasis on topics used for the materials processing area, particularly from the author's research interests. Chapter 1, "Introduction to Fluid Mechanics," briefly describes the common terms used in fluid mechanics. Chapter 2, "Fluid Statics," covers the usual topics with an interesting section on fluid interface problems. Chapter 3, "Mass, Energy and Momentum Balances," would be more aptly described as a control volume approach to modeling fluid flow problems. Chapter 4, "Viscous Flow and Friction: Confined, Open, Free Stream and Porous Media Flows," covers a variety of problems including friction coefficients, atomization, compressible flow, and pumps. The first four chapters comprise what could be called the "macroscopic" view of fluid mechanics. The next section of the book develops the detailed flow structure equations used on a variety of example problems. The chapter on "Introduction to Differential Fluid Mechanics" uses a shell balance approach to introduce some basic laminar flow equations for simple geometries. Chapter 6, "Unidirectional Flows," examines basic one-dimensional flow situations: radial, slit and tube flows. Chapter 7, "Two-Dimensional Laminar Flows: Creeping, Potential, and Boundary Layer Flows," covers a broad spectrum of problems that show relevant simplifications of the Navier-Stokes equations. The rest of the book is application-oriented starting with chapter 8, "Nearly Unidirectional Flows: Lubrication and Stretching Flows," that covers the lubrication approximation as applied to various material processing problems. Fiber spinning is used as an example to develop equations suitable for stretching flows. Chapter 9, "Rheology and Flows of Non-Newtonian Liquids," discusses various constitutive equations for generalized Newtonian fluids, and there is some introductory material on viscoelastic fluids. Finally, "Turbulent Flow and Mixing" is a brief chapter that introduces the time-averaged flow equations, some empirical mixing models for tanks and some discussion on laminar stretching of fluid elements.

The book is not easy to read. Text material is covered at an advanced undergraduate mathematical level. The discussion is uneven with some topics getting a detailed treatment in the text before being exposed in a problem. Other topics are introduced within a problem solution. Topics are sometimes split and covered in an unexpected order; for example, various forms of the boundary layer equations are introduced early in the book, but the more detailed explanation is not provided until the end of Chapter 7. Some basic equations, such as the Poiseuille flow in a tube, are derived at least twice without reference to the previous derivation. Explanations of basic measurement techniques are not complete and may confuse the student. The author's terminology is sometimes different to that found in many standard texts; for example, the species or mass conservation equations are called the solute mass-transfer equations. There are many typographical errors, some of which could confuse the novice. It would have been better to use a single system of units throughout the book. The index misses some important keywords, and not all the relevant pages are provided for given topics. These difficulties with the book would make me hesitate before I would consider it as a textbook for an undergraduate fluid mechanics course or a materials processing course. The book, however, does have many interesting and challenging problems taken from practical flow situations. Some of the problems can be solved at various levels of simplification and would provide interesting projects for students.

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Numerical Methods for Problems with Moving Fronts

By Bruce A. Finlayson, Ravenna Park Publishing, Seattle, WA, 1992, 605 pp., $60.00.

Some of the most interesting systems described by sets of partial differential equations are those whose solutions display some feature that changes very quickly in a small region of space. One of the classical examples of such systems is the boundary layer of a flow field near a solid wall. Such features are even more interesting and difficult to understand when they arise from physical processes occurring within a domain rather than being forced by external boundary conditions. These systems display "moving fronts," the topic which this book addresses. Of course, in addition to their mathematical complexity, such problems are relevant to describ-